Mobility Hierarchy and Simulation of a Modular, Reconfigurable, Tetrahedral Robot System

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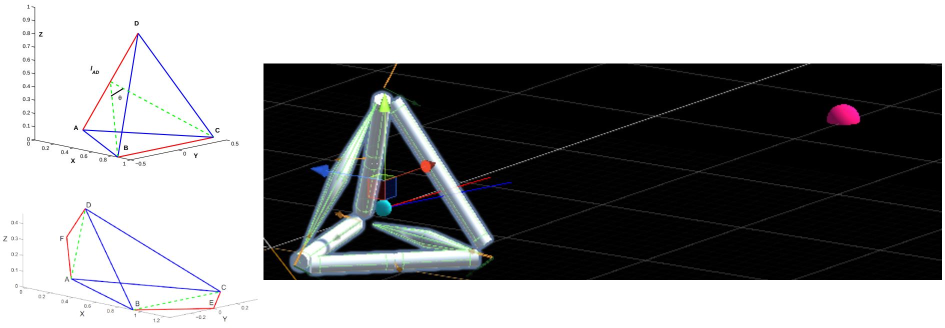
Abstract. Three dimensional modular robots are notoriously complicated to design and control. In order to make use of such a system for observation and experimentation, we have created a simulation of numerous identical tetrahedral robot modules. This paper describes the construction and physical rules surrounding the single structures as well as how one unit combines with others to create a composite. Various "small" group capabilities are discussed as well as an overview of how the simulation handles larger structures. Finally, an experimental comparison is presented where the system is used for testing the performance of these modular robots in the presence of an obstacle.

1 INTRODUCTION

Reconfigurable robot systems made up of a multitude of identical units have been of interest to the robotics community for many years [1–4]. Many different methods have been identified to control the configuration and motion of such systems. It is the goal of this paper to describe the behavior of a system made up of tetrahedral units as introduced in our previous work [5, 6].

Research in the specific arrangement and configuration of these types of systems has been conducted in two-dimensional groupings [7]. With the geometry of our tetrahedral units, it is necessary to delve into the three-dimensional domain. With this added complexity, the entire system was implemented in simulation, allowing us to test various parameters and identify their limitations. In this paper, we will examine the construction of a single unit, how it behaves alone, the formation of a physically connected group, the enhanced capabilities of the group, how it behaves relative to single units in the presence of an obstacle, and we will briefly discuss the additional difficulties that arise when multiple units are present.

The original motivation behind making our units tetrahedrons was for robustness in dispersion. With many examples from literature [8], multi-robot systems require initialization to a particular orientation or grouping before being able to perform their tasks. Our units were designed so that, should they be drop-dispersed, a single unit will be able to function regardless of how it lands.



(a)

(b)

Fig. 1: (a) Original tetrahedral units explored in previous work [5, 6]. Undeflected structure (top left), deflected structure (bottom left). (b) Physical simulation of tetrahedral unit in Unity. The magenta sphere marks the unit's mission goal.

To focus on the behavior of the single units and collective structures, these tetrahedrons were simulated with Unity [9], using the Unity Physics Engine. Our simulation uses controlled joints to assemble each unit at runtime. While the components are created in the same orientation to each other within one unit, the completed unit's orientation with respect to the ground is randomized and each one is allowed to drop onto the planar "ground" surface and settle.

For a multi-unit system like this to perform at the single unit level as well as in tandem with other, connected units, two different behaviors were implemented. The first is the movement of the single unit, covered in previous work [5, 6]. The second is the movement of a single unit when connected to a structure made up of other units. In our simulation, the second behavior is implemented at two different scales. At one extreme, with very few units attached together, each tetrahedron is allowed to actuate against the ground and the other units. At the other extreme, to represent a structure with many units connected, a single "loose" unit is simulated on a surface made up of triangular faces that represent the "skin" of a much larger multi-unit structure.

1.1 Assumptions

Within the simulation space, the tetrahedral units are created according to a normalized scale. Each edge component is assumed to be 1 unit of mass and 1 unit of length. Gravity is also simulated, with the assumption that our unit of mass is equivalent to 1 kg. A simple input is provided to each unit, a desired mission goal location in (x, y, z) coordinates.

1.2 Simulated Behavior

While it may seem like a simple task for the multi-unit system to move from one location to another, the governing behavior, described in Fig. 2, is nontrivial. In our simulation, it is expected that single units are instantiated across the entire workspace of interest. Each unit is given the desired goal location and it travels, alone, towards those coordinates. Should it fall within a certain proximity of another unit, it will connect to it and form a substructure of two or more units.

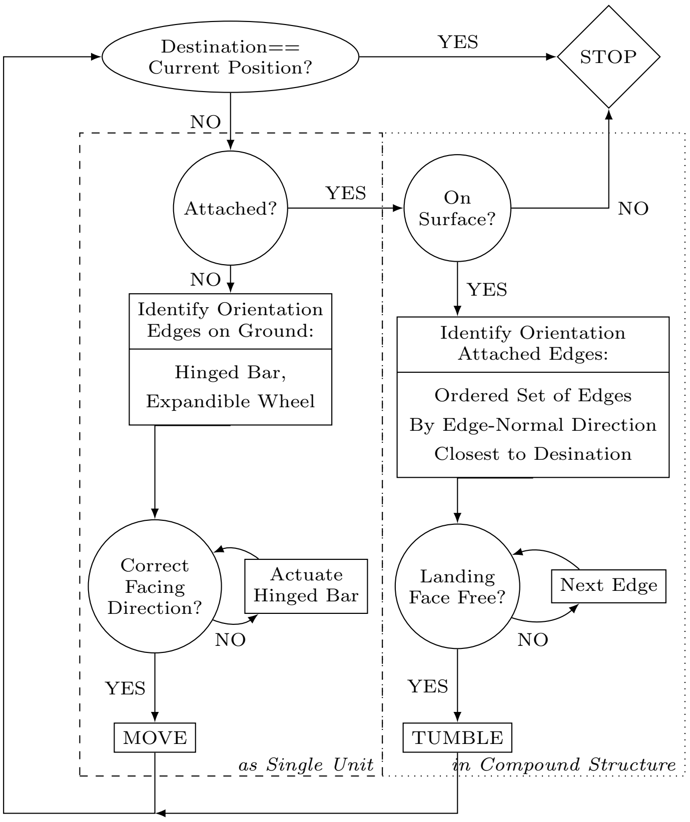


Fig. 2: Autonomous behavior of tetrahedron unit.

As multiple units get closer and closer to the goal location, they will gather in larger and larger clumps, creating a "snowball effect".

2 MOVEMENT

2.1 Single Unit

The movement of a single tetrahedral unit was chosen from the various mobility modes whose kinematics were described in our earlier work (Fig. 1a). To maintain structure as well as flexibility and movement speed, the following configuration was chosen (Fig. 3). The six sides of the unit tetrahedron are made up of three pairs of components, with each component situated across from its partner such that no component is adjacent to an identical one.

The first pair are static bars. They do not deform and are attached to their neighbors at each end with ball joints. The second pair are hinged bars. They are identical to the static bars except they are allowed to bend in the middle, enabling a single unit to change its height and planar orientation/heading. The third pair are expandable wheels [10–13]. These are primarily responsible for moving a single unit across the ground plane. Like the static edge components, each vertex of the tetrahedron unit is coded with three sets of ball joints, allowing

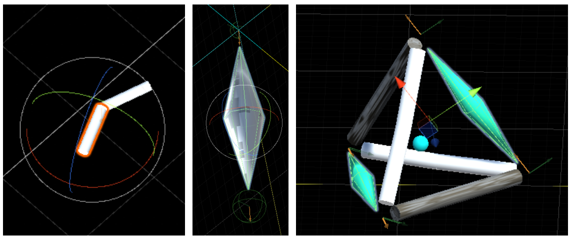


Fig. 3: [Left] A hinged bar. [Center] An expandable wheel. [Right] A single unit tetrahedron with static bar edges (textured grey), hinged bar edges (white), and expanded wheels (teal with diamond silhouette). Ball Joint constraints can be seen on both the expandable wheel and the complete unit as orange and green arrows.

free or actuated rotation between the adjacent edges. The rotational constraints can also be seen in Fig. 3

It is outside the scope of this study to specify compatible motors and other prototype components required to build each tetrahedron unit. It is, however, important to note that the dynamics of the system will be highly dependent on the final dimensions of the units that are built. It will be important to optimize the materials used as well as the power of the motors and actuators while keeping in mind that these single units will be connected to others and may be required to bear a much higher load than that of a single unit moving alone.

2.2 Multiple Units

As mentioned in the previous section, when two or more units are within a certain distance, they will form a connection. In our simulation, a simple co-location of two vertices was implemented and the two closest faces are then merged. In the physical implementation, it will be up to the user how to design the attaching mechanism(s). Because the faces of the tetrahedrons are merely empty spaces, it is the coincident edges that determine how the resulting structure will behave. Within our simulation, the edges are combined in Rochambeau fashion as shown in Table 1. The resultant combination of three units can be seen in Fig. 4.

Edge 1	Edge 2	Merged Result
Static Edge	*	Static Edge
Expand Wheel	Hinged Bar	Expand Wheel
Expand Wheel	Expand Wheel	Expand Wheel
Hinged Bar	*	*

Table 1: Edge Merging Rules

It is notable that Fig. 4 [Right] also highlights additional, non-physical components that are necessary for organization and identification of constraints. The set of components shown in green wireframe belongs to the unit tetrahedron in the middle. It is composed of one large sphere collider and four smaller ones. (The fourth smaller collider is beneath the ground plane and may be difficult to see.) The large sphere collider serves as a proximity sensor for detecting when other units are close. The smaller colliders are each situated normal to a face of

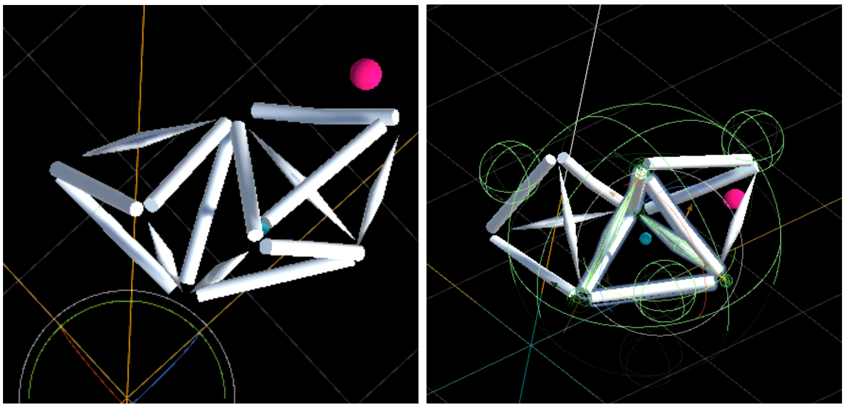


Fig. 4: A compound structure made up of three tetrahedral units. [Left] Top view, [Right] Oblique view. The teal marker shows the centroid of the middle unit and the magenta marker indicates the destination for this structure.

the central tetrahedral unit. They allow the unit to detect if each face is free for rotational actuation.

When multiple tetrahedral units are connected, there are two options for mobility. One is to use the same mechanism as the single unit, the expandable wheel, to pull the entire structure across the ground. The second is to allow the connected units to move against each other so that the overall orientation shift changes the Center of Gravity (CG) location of the entire structure. With the first option, because of the way the units are constructed, we know that each downward face must have at least one expandable wheel on it. Given randomized edge merging, there is a $\frac{1}{3}$ chance that the expandable wheel will become a static edge, unable to articulate or contribute to the movement.

Consider the relatively small compound structure in Fig. 4. Two of the units have expandable wheel edges still touching the ground. The unit on the right is not touching the ground at all. For a large group of connected units that may be many units tall, only the bottom layer of n units are in contact with the ground. Of the bottom layer, $\frac{1}{3}$ of edges started out as expandable wheels. Aside from the perimeter edges, the number of merged edges is only half of $3n$. Of the edges that started as expandable wheels, is it expected that $\frac{1}{3}$ of them merged with static edges, $\frac{1}{3}$ of them merged with other expandable wheels, and $\frac{1}{3}$ of them merged with hinged bars. Accordingly, the resulting number of expandable wheels is:

$$n * \frac{1}{3} * \frac{1}{2} + n * \frac{1}{3} = n * \frac{1}{2} \tag{1}$$

And similarly, the resulting number of hinged bar edges is:

$$n * \frac{1}{3} * \frac{1}{2} = n * \frac{1}{6} \tag{2}$$

Recall that, with a single unit, the hinged bar edge components are used to adjust the height and the planar orientation of the unit. In a connected group, it is expected that there will not be enough hinged bar elements to allow all of the expandable wheel edges to point in the desired directions of motion. This does not consider the related problem of physical constraints that arise from the multi-unit connected mesh, and the likelihood that fewer than desired expandable wheels and hinged bar elements remain available for use.

Mobility Within a Conjoined Multi-robot Group. There are many different ways of mitigating the over-constrained states in a connected group to enable lateral movement of the whole. For example, in Fig. 4, the leftmost connected tetrahedral unit is only connected to the middle unit by an edge, as opposed to the rightmost unit which is connected by an entire tetrahedral face. This allows the left unit to retain more ground contacts and contribute more to the system's overall mobility. For the purposes of our investigation, however, we will consider a different approach to the problem: allowing individual units to move along the surface of the multi-robot group.

The first step to allowing inter-unit movement is determining if the unit in question is on the outer surface of the connected structure. This determination is necessary assuming the bulk of the group is packed "tightly" with no large inner cavities. For this determination, a ray-casting method is used where a ray, originating from the group CG, is cast towards the unit CG. If a cone angle 60 degree sweep of the ray encounters nothing within 1 edge length, then the unit is considered to be in the outer "skin" layer of the group and will be allowed to move (see Fig. 5). Alternatively, as mentioned previously, Unity enables us to use colliders to detect allowable rotational directions.

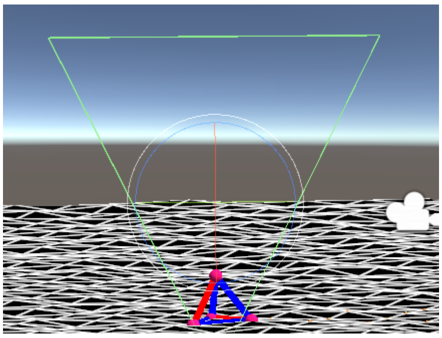


Fig. 5: A ray-cast cone to determine if a tetrahedral unit is on the surface of a compound structure.

In constructing our simulation, both methods were tried and it was observed that the main determining factor for which method is superior is the number of units within a compound structure. For a small number of units (≤ 10), the colliders work best, as they not only determine if a unit is allowed to move, but they also identify the faces that are free and subsequently, the edges about which the unit can rotate. For larger numbers of units, the ray-casting method is more beneficial in that it requires less data overhead to determine if a unit is in the "skin" layer. There are numerous ways by which both methods can be adjusted for optimal performance.

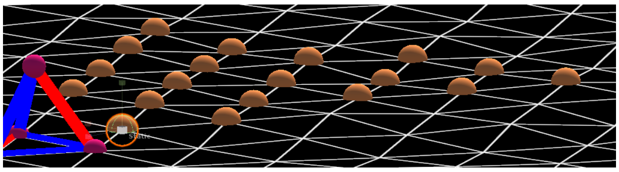
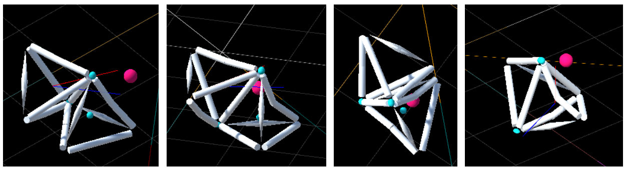


Fig. 6: A single tetrahedron unit travelling along the surface of a connected group. Orange spheres indicate the vertices within a ray-cast sweep cone that make up the navigation mesh for the single unit to move toward its destination.



(a) (b) (c) (d)

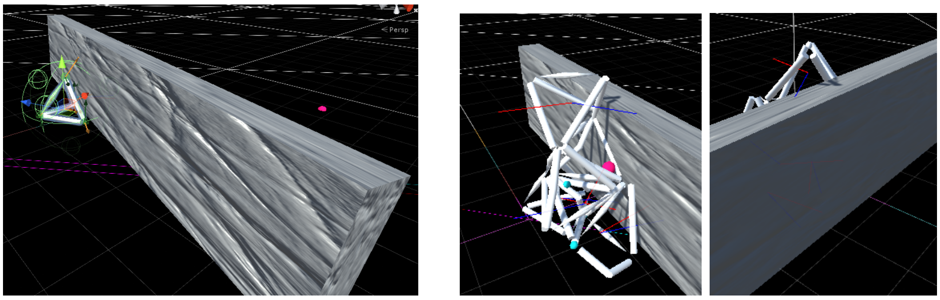
Fig. 7: Tumbling motion of single tetrahedron unit as it climbs across an identical unit. (a) Two connected units initially release two of their merged edges and start to rotate around the remaining shared edge. (b) The left unit rotates further while the right unit remains almost stationary. (c) The far face of the two units come together. This image is taken from above. (d) The coincident edges merge according to their types and they are ready to begin the cycle again.

Once a tetrahedral unit self-determines that it is in the "skin" layer, it can contribute to inter-unit movement behavior. Whether a unit is connected to a group structure or is acting alone, its goal is the same, to move its current CG to the goal location. Unlike when acting alone, a "skin" unit exists in a completely known environment: the surface mesh created by all of the other units of its group. The only places it can go are on the navigation mesh made up of the vertices of the other units. Using the local surface normal and a ray-cast toward the goal location, the moving unit can identify a small subset of vertices that form a path toward its goal. Our simulation uses a meshed surface to represent the bulk structure in order to examine this behavior (illustrated in Fig. 6).

To move, a tetrahedral unit tumbles toward one of the nearest vertices/nodes in the navigation mesh, releasing/unmerging two base edges and merging two side edges as they come down to meet the mesh surface, as shown with two units in Fig. 7. In addition, the moving unit may find it necessary to navigate toward a node that is not captured in the navigation mesh, in order to reach a node in the navigation mesh. Any node that is adjacent to a base node of the moving unit is at a maximum of two moves away. This method allows for multiple units to move simultaneously, possibly crossing each other's paths and treating each other as additional navigation nodes.

3 ENVIRONMENT WITH OBSTACLE

One particularly significant use case for a multi-unit connected body is in an environment with obstacles (see Fig. 8a). For our single unit structure, inclined slopes of up to 20 degrees are perfectly manageable. When it comes to walls and other vertical obstacles, however, the single unit is generally unable to surmount anything more than 0.5 units in height from the flat ground plane. Out of 100 samples, fewer than 10 were able to surmount an obstacle of this height. From examination of the units that were observed traversing the obstacle, most can be discarded as exhibiting aberrant behavior, such as vibrational abnormalities between rigid bodies, and polygon collision issues.



(a)

(b)

Fig. 8: (a) A single unit tetrahedron is stopped by a wall 2 units in height. Graphical texture of the wall is only applied for visual differentiation. The surface is modeled as a smooth plane. (b) A multi-unit structure of large enough number can easily "reach" over a vertical obstacle.

For a compound structure, vertical obstacles are less of a problem. Because multiple units can build themselves into taller structures, they are able to "reach" over any vertical obstacle given enough units. Because of the process by which a set of single units connect together, in the order in which they happen to meet, to form a conjoined multi-robot system, a connected structure that should be able to traverse a vertical obstacle may fail due to over-balancing and malformation. When some units surmount the apex of a wall, they may also detach from the compound structure and strike off on their own, leaving a number of others behind, potentially too few to make it over the wall.

Fig. 8b shows a group of 7 units that have made their way to the obstacle and have gathered and connected there. This image shows a number of the key elements we have mentioned. First, from the geometry of each unit, we know that 4 or 5 robots is the minimum number required to reach the top of this 2-unit wall. Second, due to their autonomous behavior and empty space within this structure, many times the connected units will fold in on themselves and interfere with one another, creating a visual jumble. In this case, the bottom of the structure is difficult to discern. Third, the bent edge protruding from the bottom of the structure is a hinged bar that is exhibiting the results of forces acting upon it from the other components. The hinge in the middle of the hinged bar is a spring joint with stiffness $k = 150\,kg/s^2$. By choosing a stiffer spring constant, the deflection becomes less pronounced and can be made to disappear completely. Values of k up to 500 kg/s^2 have been examined. Fourth, as the top unit finishes its rotation and surmounts the wall, the bulk of the compound structure is no longer in the direction of its destination. As per our algorithm, the top unit unmerges its connected edges and falls off the structure to continue on its way to its destination, leaving behind the other 6 units who are left to clamber over each other, still trying to surmount the wall.

3.1 Timed Trials

As a proof of concept to understand the benefits of allowing our tetrahedral robots to join together, simulations were run with a height 1 wall and a group of 5 robots per trial (see Fig. 9). For each trial, 5 unattached tetrahedral units were dropped at 10 unit increments apart, perpendicular to the wall. Each one had a randomized orientation when instantiated, and all units were deployed simultaneously. The goal for each group was to get one unit into the goal region, represented by the metallic grey section on the left side of Fig. 9.

Fig. 9: Top view of a timed trial of 5 tetrahedral units instantiated (nominally) at 10, 20, 30, 40, and 50 units away from the middle of the wall. The wall is 1 unit high and 10 units long.

Five trials were run without allowing the units to connect and five more were run allowing the formation of compound structures. The single units were able to identify the wall obstacle and adjusted their headings to go around it using a ray-cast toward the goal region. The compound units simply approached the nearest point of the goal region, which was the middle of the wall. The time it took each unit to arrive at the goal region is shown in Table 2.

	Single Unit Arrival Times (s)					Connectable Unit Arrival Times (s)				
Trials:	1	2	3	4	5	1	2	3	4	5
unit 1	**10.10**	**9.906**	**12.65**	**9.915**	**17.87**	**29.13**	**33.30**	**33.97**	**32.45**	**33.37**
unit 2	28.19	23.03	24.29	19.48	28.00	40.52	39.02	38.95	38.86	42.09
unit 3	31.37	29.29	30.91	35.03	29.31	48.90	52.58	49.00	49.44	48.90
unit 4	40.91	40.18	39.19	42.28	39.17	–	–	–	–	–
unit 5	54.43	52.90	49.34	49.23	58.03	–	–	–	–	–

Table 2: The time required for each tetrahedron to arrive in the goal region.

At first glance, it appears that the single units accomplished the goal in less than half of the time it took for the connectable groups. So clearly they are the winners of this trial. It is important to note, however, that this is a rather narrow wall and the single units were able to make it to either end without too much trouble. If the wall had spanned the entire field, none of the single units would have made it through. It can also be observed that the travel time of the single units varied almost linearly with the distance to the wall. This calls to question what would happen if the wall were longer.

To that end, the same trials were run for wall lengths in 10 unit increments up to 100 units in length. The results of the arrival times are depicted in Fig. 10. As the wall gets longer, the single units are required to travel further to get around the obstacle, while the connected unit groups are able to achieve the mission goal at a generally constant rate.

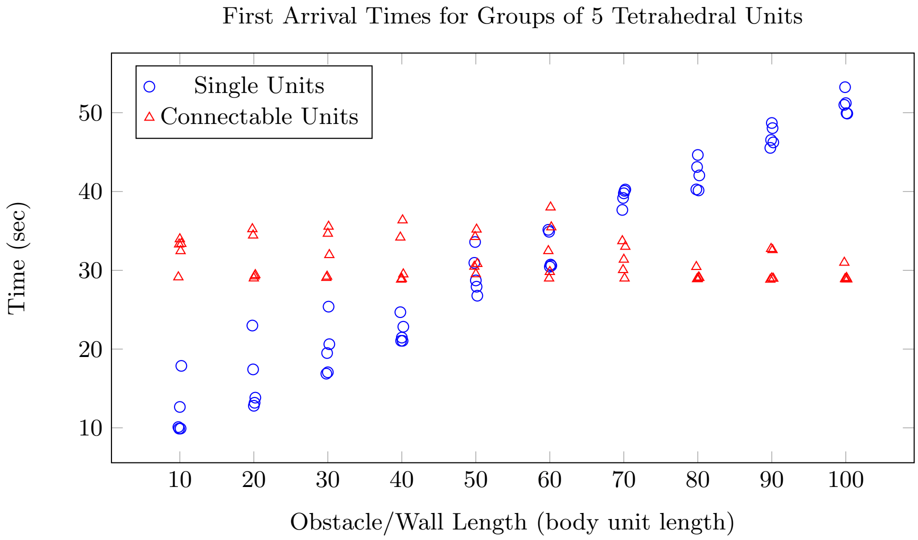


Fig. 10: First unit arrival times versus wall length.

4 SUMMARY

A few important observations can be made from the trials. The connectable units had to wait for the arrival of at least three units at the wall prior to being able to assemble into a tall-enough structure. Because of their starting positions, the units tended to all be traveling in approximately the same direction at the same time, making it difficult for them to connect and travel together in smaller groups. (From observation, this did occur but it was rare.) So, if we were to compare the arrival times of the third single unit to that of the grouped units getting one member into the goal region, the difference would be minor.

Once the initial connected “pile” was formed at the base of the wall, the stragglers who had not connected to other units prior to arriving at the wall had no trouble making it over or taking the place of previously arrived units after they surmounted the wall.

From the simulated exercises conducted with multiple unit tetrahedrons, we have determined the basic behaviors of a single unit as well as a general understanding of a compound structure and its mobility. We have also identified a prevalent use case for this type of system. In doing so, we have identified additional questions and research topics for future work. This initial study serves as a starting point for future analysis that will be conducted.

5 FUTURE WORK

For a better understanding of this system, the roles of the following key parameters in system performance can be further examined:

- Single Unit Parameters:
  - Deflection limit/actuation/spring constant for the hinged bar
  - Deflection limit/radial actuation for the expandable wheel

- Multi Unit Parameters:
  - Distance/proximity limit for forming connections between units
  - Distance tolerance for achieving goal location
  - “Skin” layer determination raycast angle
  - Navigation mesh node identification raycast angle

In addition, the physical behavior of a connected group of robots has yet to be fully analyzed. With their current behaviors, absent an obstacle, the final configuration of a group of conjoined tetrahedral robots resembles a sphere or dome with a flat bottom. The flatness or eccentricity of the final shape is entirely dependent on the vertical (z) value of the goal CG location. If the z location is set to a point above where the group of units can reach, it creates an instability where lower units try to climb over their neighbors, decreasing the base footprint and contact area to the ground plane until the whole system falls over.

Within the algorithm, there is also room for adjustment. It has yet to be established whether and in what circumstances a single unit may move faster alone than when attached to a group. The answer may well depend on selecting the correct parameters for the variables indicated above. If a single unit performs better alone, it may be desirable to keep the individual units separated. If a single unit performs better alone at certain times, it may be desirable to allow a connected group to intentionally dissolve and reconnect at a later time. The full dissolution of a group is not yet included in our simulation.

Between individuals and a single group, there is also the issue of forming multiple small groups. Varying the proximity limit for connecting vertices together, we may achieve some very different structures with different capabilities. In addition, we have neglected to fully explore the lateral movement of connected multi-robot structures in our current system. The actuation of the ground layer is a rich topic for exploration. We chose to assume a tightly packed group. A loosely packed or hollow group may be much more desirable for certain applications such as adjustable buoyancy or carrying an internal payload.

Finally, it is important to note that system-level behaviors arise unexpectedly and unintentionally. One such example happened when too may units were instantiated too quickly and too closely together. They created a tangled mess of edges. When left alone, however, they managed to untangle themselves and finally were able to reach their individual goals. This incident, along with the multi-unit, vertical obstacle traversal case where a significant portion of the units were left behind, identified the need for the creation of a cost function or attrition allowance for these types of systems. It would be interesting and useful to consider the meta parameters that can be adjusted to define how a mission goal can be judged to be successfully accomplished.

It was the purpose of this paper to document the simulation groundwork for a three dimensional, reconfigurable, multi-robot system composed of tetrahedral robots. Moving forward, we will be able to use it to select design and configuration parameters to meet different performance requirements. The simulation will be able to test our tetrahedral units with various physical and algorithmic constraints and it will enable us to stress-test the system in various adverse environments.

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